

Fermi surface truncation from thermal nematic fluctuations

Hiroyuki Yamase¹ and Walter Metzner²

¹*National Institute for Materials Science, Tsukuba 305-0047, Japan*

²*Max-Planck-Institute for Solid State Research, D-70569 Stuttgart, Germany*

(Dated: May 31, 2012)

We analyze how thermal fluctuations near a finite temperature nematic phase transition affect the spectral function $A(\mathbf{k}, \omega)$ for single-electron excitations in a two-dimensional metal. Perturbation theory yields a splitting of the quasi-particle peak with a d -wave form factor, reminiscent of a pseudogap. We present a resummation of contributions to all orders in the Gaussian fluctuation regime. Instead of a splitting, the resulting spectral function exhibits a pronounced broadening of the quasi-particle peak, which varies strongly around the Fermi surface and vanishes upon approaching the Brillouin zone diagonal. The Fermi surface obtained from a Brillouin zone plot of $A(\mathbf{k}, 0)$ seems truncated to Fermi arcs.

PACS numbers: 71.18.+y, 71.10.Hf, 71.27.+a

The concept of nematic order in interacting electron liquids has attracted considerable interest over the last decade, mostly in the context of (quasi) two-dimensional systems [1, 2]. In a nematic state an orientational symmetry of the system is spontaneously broken, without breaking however the translation invariance. One route toward a nematic state is via partial melting of stripe order in a doped antiferromagnetic Mott insulator [3]. Alternatively, a nematic state can be obtained from a Pomeranchuk [4] instability generated by forward scattering interactions in a normal metal [5, 6]. On a square lattice, the most natural candidate for a Pomeranchuk instability has $d_{x^2-y^2}$ symmetry.

Signatures of nematic order with a $d_{x^2-y^2}$ symmetry have been observed in several strongly interacting electron materials. A nematic phase with a sharply defined phase boundary has been established for $\text{Sr}_3\text{Ru}_2\text{O}_7$ in a strong magnetic field [7]. Nematic order has also been observed in the high temperature superconductor $\text{YBa}_2\text{Cu}_3\text{O}_y$ in transport experiments [8] and neutron scattering [9]. Due to the slight orthorhombicity of the CuO_2 planes one cannot expect a sharp nematic phase transition in $\text{YBa}_2\text{Cu}_3\text{O}_y$. However, the strong temperature dependence of the observed in-plane anisotropy indicates that the system develops an intrinsic electronic nematicity, which drastically enhances the in-plane anisotropy imposed by the structure [10, 11].

Nematic fluctuations close to a continuous nematic quantum phase transition naturally lead to non-Fermi liquid behavior [12–14]. For a d -wave Pomeranchuk instability on a square lattice, the decay rate of electronic excitations is strongly momentum dependent along the Fermi surface. At the quantum critical point, the decay rate for single-particle excitations is proportional to $d_{\mathbf{k}}^2 |\omega|^{2/3}$, where ω is the excitation energy and $d_{\mathbf{k}}$ is a form factor with d -wave symmetry, such as $d_{\mathbf{k}} = \cos k_x - \cos k_y$ [13, 15]. Landau quasi-particles are thus unstable everywhere on the Fermi surface except at the Brillouin zone diagonals, where $d_{\mathbf{k}}$ vanishes. The temperature depen-

dence of the decay rate in the quantum critical regime near the quantum critical point also differs strongly from Fermi liquid behavior [16, 17].

Previous works on non-Fermi liquid behavior caused by nematic fluctuations focussed on the quantum critical point and the quantum critical regime at finite temperature. In this paper we analyze the spectral function for single-particle excitations in the *thermal* fluctuation regime near a nematic phase transition at finite temperatures. We show that a perturbative calculation of the self-energy in that regime leads to a splitting of the quasi-particle peak with a d -wave form factor in the single-particle excitation spectrum, reminiscent of a pseudogap. However, in a self-consistent calculation the split peak in the spectral function is replaced by a single broad peak. In the Gaussian fluctuation regime, a summation of vertex corrections to all orders is possible and confirms the self-consistent result. The Fermi surface obtained from the peak of the spectral function at zero excitation energy is thus smeared by a smooth broadening, which is most pronounced near the points $(\pi, 0)$ and $(0, \pi)$ of the Brillouin zone, while it gradually decreases toward the Brillouin zone diagonal.

We consider a one-band system of electrons on a square lattice with a tight-binding dispersion $\epsilon_{\mathbf{k}}$ and an effective interaction of the form [13]

$$H_I = \frac{1}{2L} \sum_{\mathbf{q}} g(\mathbf{q}) n_d(\mathbf{q}) n_d(-\mathbf{q}), \quad (1)$$

where $n_d(\mathbf{q}) = \sum_{\mathbf{k}, \sigma} d_{\mathbf{k}} c_{\mathbf{k}-\mathbf{q}/2, \sigma}^\dagger c_{\mathbf{k}+\mathbf{q}/2, \sigma}$ are d -wave density fluctuation operators, and L is the number of sites. The function $g(\mathbf{q})$ is negative and peaked at $\mathbf{q} = 0$, so that forward scattering dominates. An effective interaction of the form H_I can be obtained from microscopic models such as the Hubbard or t - J model [5, 6].

For sufficiently negative values of $g = g(0)$ the interaction generates a d -wave Pomeranchuk instability leading to a nematic state with a spontaneously broken orientation symmetry [13, 18–20]. A suitable order parameter

characterizing the symmetry breaking is provided by the expectation value $\langle n_d(0) \rangle$. Close to the transition (if continuous), strong d -wave density fluctuations with a long wavelength develop, which lead to a singular effective interaction. In the quantum critical regime the effective interaction is dynamical and of the form [13, 16]

$$D_{\mathbf{k}\mathbf{k}'}(\mathbf{q}, \nu_n) = \frac{g d_{\mathbf{k}} d_{\mathbf{k}'}}{(\xi_0/\xi)^2 + \xi_0^2 |\mathbf{q}|^2 + |\nu_n|/(u|\mathbf{q}|)}, \quad (2)$$

where $\nu_n = 2\pi nT$ is a bosonic Matsubara frequency; ξ is the nematic correlation length, while ξ_0 and u are non-universal parameters determined by the momentum dependence of $g(\mathbf{q})$ and the band structure. In the thermal fluctuation regime near the finite temperature phase transition, quantum ($\nu_n \neq 0$) fluctuations are cut off by temperature, such that only the classical part of the effective interaction,

$$D_{\mathbf{k}\mathbf{k}'}(\mathbf{q}) = D_{\mathbf{k}\mathbf{k}'}(\mathbf{q}, 0) = \frac{\tilde{g} d_{\mathbf{k}} d_{\mathbf{k}'}}{\xi^{-2} + |\mathbf{q}|^2}, \quad (3)$$

with $\tilde{g} = g/\xi_0^2$, is important.

The nematic transition on a square lattice belongs to the two-dimensional Ising universality class. A thermal phase transition at a critical temperature $T_c > 0$ is possible, since the dimensionality of the system is above the lower critical dimension (one). The correlation length ξ diverges at T_c . Approaching the critical temperature, one first passes through a Gaussian fluctuation regime, where order parameter interactions are not important. Moving closer to T_c , one enters the Ginzburg region, where order parameter interactions become relevant, and the fluctuation propagator acquires an anomalous scaling dimension [21]. Close to the quantum critical point, the width of the Ginzburg region is of order $T_c/|\log T_c|$ [22].

The momentum resolved spectral function for single-particle excitations can be written as

$$A(\mathbf{k}, \omega) = -\frac{1}{\pi} \text{Im} G(\mathbf{k}, \omega) = -\frac{1}{\pi} \text{Im} \frac{1}{\omega - (\epsilon_{\mathbf{k}} - \mu) - \Sigma(\mathbf{k}, \omega)}, \quad (4)$$

where $G(\mathbf{k}, \omega)$ and $\Sigma(\mathbf{k}, \omega)$ are the retarded Green function and self-energy, respectively. We first compute the self-energy perturbatively to first order in the effective interaction. The contribution from classical fluctuations is given by [16]

$$\Sigma(\mathbf{k}, \omega) = -T \int \frac{d^2 q}{(2\pi)^2} D_{\mathbf{k}\mathbf{k}}(\mathbf{q}) G(\mathbf{k} - \mathbf{q}, \omega). \quad (5)$$

In a non-selfconsistent evaluation of Eq. (5) one approximates G by the non-interacting Green function $G_0(\mathbf{k}, \omega) = [\omega - (\epsilon_{\mathbf{k}} - \mu) + i0^+]^{-1}$. The self-energy can then be computed analytically. The imaginary part has been obtained already previously [16]. For momenta close to the Fermi surface and small frequencies one finds

$$\text{Im} \Sigma(\mathbf{k}, \omega) = \frac{\tilde{g} d_{\mathbf{k}}^2}{4v_{\mathbf{k}}} T \xi l(\kappa), \quad (6)$$

where $v_{\mathbf{k}} = |\nabla \epsilon_{\mathbf{k}}|$ is the velocity of the electrons, $\kappa = [\omega - (\epsilon_{\mathbf{k}} - \mu)]\xi/v_{\mathbf{k}}$, and $l(\kappa) = (1 + \kappa^2)^{-1/2}$. We assume that the Fermi surface does not cross van Hove points, such that $v_{\mathbf{k}}$ is finite. The real part of the self energy is obtained either by a direct evaluation of Eq. (5) or by a Kramers-Kronig transformation of the imaginary part as

$$\text{Re} \Sigma(\mathbf{k}, \omega) = \frac{\tilde{g} d_{\mathbf{k}}^2}{4\pi v_{\mathbf{k}}} \xi T l(\kappa) \ln \left| \frac{1 - \kappa l(\kappa)}{1 + \kappa l(\kappa)} \right|. \quad (7)$$

In Fig. 1 we show results for the spectral function as obtained from the non-selfconsistent first order calculation of the self-energy. Here and in all further numerical results we have chosen a dispersion $\epsilon_{\mathbf{k}} = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y - 2t''(\cos 2k_x + \cos 2k_y)$ with hopping amplitudes $t = 1$, $t' = -0.3$, and $t'' = 0.15$. The lattice constant is one, and the chemical potential μ has been chosen such that the electron density is fixed at $n = 0.9$. The corresponding Fermi surface is closed around (π, π) . The coupling constant is $\tilde{g} = -1.2$ and the temperature $T = 0.15$. We have not attempted to compute the correlation length, since it depends on model details such as the momentum dependence of $g(\mathbf{q})$. Instead we show results for various choices of ξ . One can see that a pronounced splitting of the quasi-particle peak develops for increasing ξ , which could be interpreted as a fluctuation precursor of the symmetry-broken state. However, we now show that the splitting disappears in a self-consistent calculation, and it is not restored by vertex corrections, at least in the Gaussian fluctuation regime.

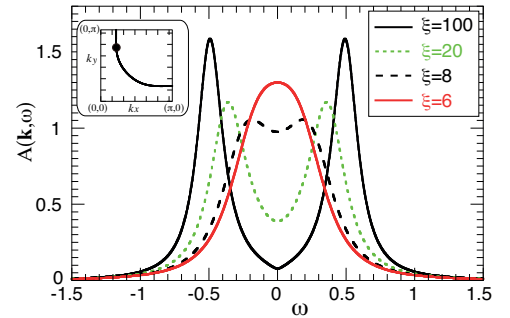


FIG. 1: (Color online) Spectral function $A(\mathbf{k}, \omega)$ as obtained from a non-selfconsistent perturbative calculation of the self-energy for various choices of the correlation length ξ . The graphs show $A(\mathbf{k}, \omega)$ as a function of ω for a fixed momentum $\mathbf{k} = (0.69, 2.34)$ on the Fermi surface remote from the Brillouin zone diagonal (see inset).

In a self-consistent evaluation of Eq. (5), with the interacting Green function on the right hand side, one has to solve an integral equation. The problem can be simplified by decomposing the momentum transfer \mathbf{q} in components q_r and q_t normal and tangential to the Fermi surface, respectively [16]. The dependence on q_t can be neglected in the momentum argument of G , such that the q_t -integral

acts only on the fluctuation propagator $D_{\mathbf{k}\mathbf{k}}(\mathbf{q})$, yielding

$$\bar{D}_{\mathbf{k}}(q_r) = \int \frac{dq_t}{2\pi} D_{\mathbf{k}\mathbf{k}}(\mathbf{q}) = \frac{\tilde{g}d_{\mathbf{k}_F}^2}{2\sqrt{\xi^{-2} + q_r^2}}. \quad (8)$$

The momentum dependence of the self-energy $\Sigma(\mathbf{k}, \omega)$ can be parametrized by the Fermi momentum \mathbf{k}_F closest to \mathbf{k} and the oriented distance from the Fermi surface k_r , which carries the sign of $\xi_{\mathbf{k}}$. One then obtains the one-dimensional integral equation

$$\begin{aligned} \Sigma(k_r, \omega) &= \frac{\tilde{g}d_{\mathbf{k}_F}^2 T}{4\pi} \int \frac{dq_r}{\sqrt{\xi^{-2} + q_r^2}} \\ &\times \frac{1}{v_{\mathbf{k}_F}(k_r - q_r) - \omega + \Sigma(k_r - q_r, \omega)}. \end{aligned} \quad (9)$$

Note that Σ does not depend on k_r and ω independently, but only on the difference $\omega - v_{\mathbf{k}_F}k_r$. The dependence of Σ on \mathbf{k}_F enters only parametrically via $v_{\mathbf{k}_F}$ and $d_{\mathbf{k}_F}$ and has not been written explicitly. The integral equation (9) can be solved numerically. The results for $A(\mathbf{k}, \omega)$ differ strongly from those suggested by non-selfconsistent perturbation theory. The quasi-particle splitting observed in the perturbative calculation (Fig. 1) is wiped out completely by the self-energy feedback into G . The spectral function exhibits only a single peak with a maximum at $\omega = v_{\mathbf{k}_F}k_r$, even for a very large correlation length ξ .

We now consider higher order contributions not contained in the self-consistent one-loop approximation (5). The sum over all self-energy contributions generated by thermal fluctuations can be written as

$$\begin{aligned} \Sigma(\mathbf{k}, \omega) &= -T \int \frac{d^2q}{(2\pi)^2} D_{\mathbf{k}\mathbf{k}}(\mathbf{q}) G(\mathbf{k} - \mathbf{q}, \omega) \\ &\times \Lambda(\mathbf{k} - \mathbf{q}/2, \omega; \mathbf{q}, 0), \end{aligned} \quad (10)$$

where $\Lambda(\mathbf{k}, \omega; \mathbf{q}, \nu)$ is the irreducible charge vertex including all vertex corrections. We exploit the fact that dominant contributions are due to small momentum transfers \mathbf{q} of order ξ^{-1} , to resum vertex corrections via an asymptotic Ward identity [23, 24]. For small \mathbf{q} , the charge vertex is related to the current vertex $\mathbf{\Lambda}$ and the propagator via the Ward identity $\nu\Lambda(\mathbf{k}, \omega; \mathbf{q}, \nu) - \mathbf{q} \cdot \mathbf{\Lambda}(\mathbf{k}, \omega; \mathbf{q}, \nu) = G^{-1}(\mathbf{k} + \mathbf{q}/2, \omega + \nu/2) - G^{-1}(\mathbf{k} - \mathbf{q}/2, \omega - \nu/2)$. In the Gaussian fluctuation regime, interactions between order parameter fluctuations are not important. In a diagrammatic representation of perturbation theory, these interactions are generated by fermionic loops with more than two vertices. Neglecting Feynman diagrams with such loops leads to two simplifications. First, the effective interaction (3) remains unrenormalized. Second, diagrams contributing to the charge and current vertices involve only an open fermionic line. Since contributions with small \mathbf{q} dominate, the electron velocity $\mathbf{v}_{\mathbf{k}}$ entering the current operator is almost conserved such that the current vertex can be expressed by the charge vertex as $\mathbf{\Lambda}(\mathbf{k}, \omega; \mathbf{q}, \nu) = \mathbf{v}_{\mathbf{k}}\Lambda(\mathbf{k}, \omega; \mathbf{q}, \nu)$. The latter relation

holds for each Feynman diagram without fermionic loops. Combining this with the Ward identity one obtains, in the static limit $\nu = 0$,

$$\Lambda(\mathbf{k}, \omega; \mathbf{q}, 0) = \frac{G^{-1}(\mathbf{k} - \mathbf{q}/2, \omega) - G^{-1}(\mathbf{k} + \mathbf{q}/2, \omega)}{\mathbf{v}_{\mathbf{k}} \cdot \mathbf{q}}. \quad (11)$$

Inserting Eq. (11) into Eq. (10), and using the Dyson equation $G^{-1} = G_0^{-1} - \Sigma$, one obtains a closed system of equations for Σ and G . Decomposing the momenta \mathbf{k} and \mathbf{q} in radial and tangential components, and integrating $D_{\mathbf{k}\mathbf{k}}(\mathbf{q})$ over q_t as before (self-consistent solution), one finds a one-dimensional linear integral equation for G ,

$$\begin{aligned} (\omega - v_{\mathbf{k}_F}k_r + i0^+) G(k_r, \omega) &= \\ 1 + T \int \frac{dq_r}{2\pi} \frac{\bar{D}_{\mathbf{k}_F}(q_r)}{v_{\mathbf{k}_F}q_r} G(k_r - q_r, \omega), \end{aligned} \quad (12)$$

with $\bar{D}_{\mathbf{k}_F}(q_r)$ from Eq. (8).

Note that vertex corrections cannot be summed by the above method at a nematic quantum critical point or for the related problem of non-relativistic fermions coupled to a $U(1)$ gauge field. This is because in these cases the dominant momentum transfers are almost tangential to the Fermi surface, so that the term $\mathbf{v}_{\mathbf{k}} \cdot \mathbf{q}$ becomes subleading compared to contributions originating from fluctuations of the electron velocity [24].

The integral equation (12) can be converted to a linear differential equation by a Fourier transformation. The differential equation can be solved by standard methods. The result for the spectral function reads

$$A(\mathbf{k}, \omega) = \int_{-\infty}^{\infty} dx \hat{A}(x) e^{i(\omega - v_{\mathbf{k}_F}k_r)x/v_{\mathbf{k}_F}}, \quad (13)$$

where

$$\hat{A}(x) = \frac{1}{2\pi v_{\mathbf{k}_F}} \exp \left[\int_0^x dx' \int_0^{x'} dx'' T \frac{\tilde{g}d_{\mathbf{k}_F}^2}{2\pi v_{\mathbf{k}_F}^2} K_0(x''/\xi) \right]. \quad (14)$$

K_0 is a modified Bessel function. Note that $A(\mathbf{k}, \omega)$ depends on k_r and ω only via the difference $\omega - v_{\mathbf{k}_F}k_r$.

In Fig. 2 we plot the spectral function $A(\mathbf{k}, \omega)$ for the same parameters as in Fig. 1. The function exhibits only a single peak with no trace of a splitting. The splitting present in Fig. 1 is therefore an artefact of the perturbation expansion, at least in the Gaussian regime. Vertex corrections do not change the self-consistent one-loop result qualitatively. Quantitatively they tend to sharpen the peak in $A(\mathbf{k}, \omega)$, but only moderately. For a large correlation length ξ the width of the peak in $A(\mathbf{k}, \omega)$ is proportional to $\sqrt{\log \xi}$, corresponding to a peak in the imaginary part of the self-energy $\text{Im}\Sigma(\mathbf{k}, \omega) \propto \sqrt{\log \xi}$ at $\omega = v_{\mathbf{k}_F}k_r$. The width of the peak in $\text{Im}\Sigma$ is also proportional to $\sqrt{\log \xi}$ and therefore increases with ξ . This is very different from the perturbative result for $\text{Im}\Sigma$, Eq. (6), where the height of the peak increases rapidly with ξ , while its width shrinks.

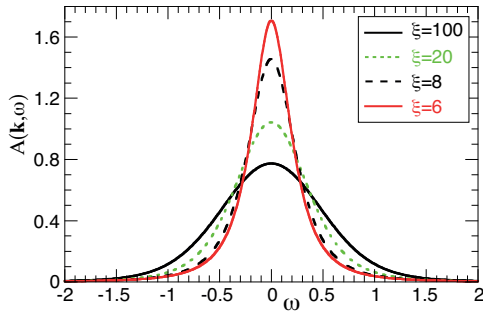


FIG. 2: (Color online) Spectral function $A(\mathbf{k}, \omega)$ as obtained from a non-perturbative resummation of contributions from thermal fluctuations, including vertex corrections. The choice of \mathbf{k} and the model parameters are the same as in Fig. 1.

In the quantum critical regime studied previously [16] the spectral function also exhibits a single peak with a temperature dependent broadening. In that regime the width of the peak is proportional to $T\xi$, with a correlation length ξ diverging as $(T|\log T|)^{-1/2}$ upon approaching the quantum critical point at $T = 0$.

It is striking that perturbation theory indicates a fluctuation precursor of the symmetry broken state at leading order, which is however not robust with respect to higher order contributions. One may compare with the case of a charge density wave with a finite wave vector, where symmetry breaking opens a gap. Perturbation theory indicates a pseudogap above the transition temperature in such systems, for example for the flux order studied in Ref. [25]. It would be interesting to analyze the fate of the pseudogap in such systems in a calculation beyond perturbation theory.

Due to the d -wave form factor in the effective interaction, the broadening of the spectral function varies strongly in momentum space. To illustrate this, we plot an intensity map of $A(\mathbf{k}, 0)$ in the first quarter of the Brillouin zone in Fig. 3. $A(\mathbf{k}, 0)$ as obtained from Eq. (13) diverges on the Brillouin zone diagonal, since $d_{\mathbf{k}}$ vanishes there. In a more complete model, where other interaction channels should be added, this divergence will be cut off at least by regular (Fermi liquid) contributions to $\text{Im}\Sigma$ of order T^2 . We have therefore included such a regular contribution ($-T^2$). Due to the rapid increase of the broadening of the peak in $A(\mathbf{k}, 0)$ upon moving away from the Brillouin zone diagonal, and the corresponding decrease in the peak height, the Fermi surface seems truncated to arcs, albeit with fuzzy ends.

In summary, we have computed the spectral function $A(\mathbf{k}, \omega)$ for single electron excitations in the presence of critical fluctuations near a thermal nematic phase transition in a two-dimensional metal. Leading order perturbation theory indicates a splitting of the quasi-particle peak. However, a resummation of contributions to all orders reveals that the splitting is an artefact of pertur-

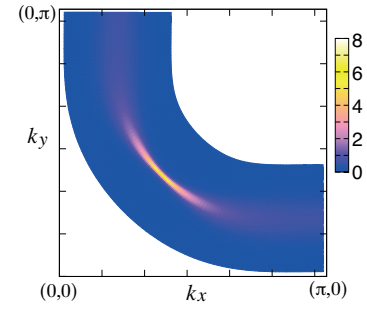


FIG. 3: (Color online) Intensity plot of the spectral function $A(\mathbf{k}, 0)$ as a function of \mathbf{k} in the first quarter of the Brillouin zone at $T = 0.2$, for a correlation length $\xi = 15$. The other parameters are the same as in Figs. 1 and 2.

bation theory, at least in the Gaussian fluctuation regime. The spectral function exhibits a pronounced broadening with a d -wave form factor, leading to features reminiscent of Fermi arcs in the Brillouin zone. The qualitative shape of the spectral function does not depend on the specific choice of parameters. Away from the Brillouin zone diagonal, the imaginary part of the self-energy has a peak at $\omega = \epsilon_{\mathbf{k}} - \mu$, in clear contrast to the conventional Fermi liquid form.

It is remarkable that the effect of Gaussian thermal fluctuations on electronic excitations could be treated non-perturbatively. The method used to sum contributions from thermal fluctuations to all orders is not restricted to nematic fluctuations, but could be applied equally well to other critical thermal fluctuations with a small wave vector, for example, close to a structural phase transition.

A continuous finite temperature phase transition is well established at the roof of the nematic dome found for $\text{Sr}_3\text{Ru}_2\text{O}_7$ in a strong magnetic field [7]. That system thus provides an opportunity to observe the fluctuation effects computed in this work, but also an experimental challenge, since the standard tool to measure the momentum resolved spectral function, that is, photoemission, is hampered by the magnetic field.

Fermi arcs have been observed in photoemission measurements of the spectral function $A(\mathbf{k}, \omega)$ in various high- T_c cuprate compounds [26]. However, the Fermi surface truncation in these materials is associated with a pseudogap formation, while we obtain only a strongly momentum dependent broadening of the spectral function. Although experiments indicate electronic nematicity at least in some cuprates [1], our results thus show that another mechanism needs to be invoked to explain the photoemission data.

We are grateful to A. Chubukov, A. Greco, A. Katanin, B. Obert, and A. Rosch for valuable discussions, and to J. Bauer for a critical reading of the manuscript.

-
- [1] E. Fradkin et al., *Annu. Rev. Condens. Matter Phys.* **1**, 153 (2010).
 - [2] M. Vojta, *Adv. Phys.* **58**, 699 (2009).
 - [3] S. A. Kivelson, E. Fradkin, and V. J. Emery, *Nature (London)* **393**, 550 (1998).
 - [4] I. J. Pomeranchuk, *Sov. Phys. JETP* **8**, 361 (1959).
 - [5] H. Yamase and H. Kohno, *J. Phys. Soc. Jpn.* **69**, 332 (2000); *ibid.* **69**, 2151 (2000).
 - [6] C. J. Halboth and W. Metzner, *Phys. Rev. Lett.* **85**, 5162 (2000).
 - [7] S. A. Grigera et al., *Science* **306**, 1154 (2004); R. A. Borzi et al., *Science* **315**, 214 (2007); A. W. Rost et al., *Science* **325**, 1360 (2009).
 - [8] R. Daou et al., *Nature (London)* **463**, 519 (2010).
 - [9] V. Hinkov et al., *Nature (London)* **430**, 650 (2004); V. Hinkov et al., *Nat. Phys.* **3**, 780 (2007); V. Hinkov et al., *Science* **319**, 597 (2008).
 - [10] A. Hackl and M. Vojta, *Phys. Rev. B* **80**, 220514(R) (2009).
 - [11] H. Yamase and W. Metzner, *Phys. Rev. B* **73**, 214517 (2006); H. Yamase, *Phys. Rev. B* **79**, 052501 (2009).
 - [12] V. Oganesyan, S. A. Kivelson, and E. Fradkin, *Phys. Rev. B* **64**, 195109 (2001).
 - [13] W. Metzner, D. Rohe, and S. Andergassen, *Phys. Rev. Lett.* **91**, 066402 (2003).
 - [14] M. Garst and A. V. Chubukov, *Phys. Rev. B* **81**, 235105 (2010).
 - [15] Recent calculations indicate that the decay rate exponent $2/3$ is slightly changed by higher order corrections, see M. A. Metlitski and S. Sachdev, *Phys. Rev. B* **82**, 075127 (2010).
 - [16] L. Dell'Anna and W. Metzner, *Phys. Rev. B* **73**, 045127 (2006).
 - [17] L. Dell'Anna and W. Metzner, *Phys. Rev. Lett.* **98**, 136402 (2007); *ibid.* **103**, 159904 (E) (2009).
 - [18] H.-Y. Kee, E.H. Kim, and C.-H. Chung, *Phys. Rev. B* **68**, 245109 (2003).
 - [19] I. Khavkine et al., *Phys. Rev. B* **70**, 155110 (2004).
 - [20] H. Yamase, V. Oganesyan, and W. Metzner, *Phys. Rev. B* **72**, 035114 (2005).
 - [21] L. Onsager, *Phys. Rev.* **65**, 117 (1944).
 - [22] J. Bauer, P. Jakubczyk, and W. Metzner, *Phys. Rev. B* **84**, 075122 (2011).
 - [23] C. Castellani, C. Di Castro, and W. Metzner, *Phys. Rev. Lett.* **72**, 316 (1994).
 - [24] W. Metzner, C. Castellani, and C. Di Castro, *Adv. Phys.* **47**, 317 (1998).
 - [25] A. Greco, *Phys. Rev. Lett.* **103**, 217001 (2009); M. Bejas et al., *Phys. Rev. B* **83**, 014514 (2011).
 - [26] A. Damascelli, Z. Hussain, and Z.-X. Shen, *Rev. Mod. Phys.* **75**, 473 (2003).